

A P P E N D I X.

CERTAIN portions of electrical science have recently come into considerable prominence, and, as they are hardly satisfactorily treated in text-books yet, it may be a help to students to say something about them here in less popular language than in the body of the book.

Electro-magnetism.

(a) The fundamental fact of electro-magnetism, ascertained by direct experiment, is that a circuit conveying a current exactly imitates a magnet of definite moment, the equivalent moment being

$$ml = \mu nAC,$$

where A is the mean area of the coil, n the number of turns of wire, C the current, and μ a constant characteristic of the medium inside the coil, whose absolute value we have as yet no means of ascertaining (§§ 68, 69, 127).

Magnetic Induction, Reluctance, and Permeability.

(b) The intensity of magnetic field at a distance r from a pole of strength m is $\frac{m}{r^2}$, and this may be called the number of lines of force (or tubes if the idea be preferred) per unit area. The total number of lines of force through a spherical surface of this radius is $\frac{m}{r^2} \times 4\pi r^2$, or $4\pi m$.

This number must likewise thread any closed surface whatever inclosing the pole; and in fact it is the number the pole possesses. It may be called the total magnetic flux or displacement, or the total magnetic induction, due to the pole; the name "induction," first used vaguely in the sense of *influence* by Faraday, having been given this definite connotation by Maxwell. The same expression likewise gives the number of lines of force due to a complete magnet; for the superposition of lines due to an equal opposite pole curves the original lines but alters not their number. With two detached poles the lines simply go from one to the other. With a complete magnet the lines all form closed loops extending from north to south through air, and back through steel. In the case of a coil they likewise are closed loops, all threading the coil and then spreading out through the surrounding medium. In all real cases, therefore, the lines of force form closed curves. Magnetic circuits are always closed, just as electric circuits are.

Take the simplest case of an anchor-ring coil, a helix bent into a closed circuit (like Fig. 47 or 29): all its lines are then inside it, and their total number, being $4\pi m$, is $\frac{4\pi\mu nAC}{l}$; where l is the mean circumference of the anchor-ring, or length of the magnetic circuit. This is called the total flux of magnetic induction, or briefly the total induction, and we will denote it by I .

Now, in the analogous case of a voltaic circuit, the current is ratio of electromotive force to resistance, and the resistance may be written $\frac{l}{\kappa A}$; κ being specific conductivity, and A sectional area of conductor of length l .

To bring out the analogy, we shall write the magnetic flux—

$$I = \frac{4\pi nC}{\mu A},$$

where the numerator is sometimes called magneto-motive force,

and the denominator magnetic resistance, or preferably, as suggested by Mr. Heaviside, magnetic *reluctance*. Obviously μ takes the place of electric conductivity, and is a sort of magnetic conductivity: it was from this point of view that Sir W. Thomson long ago christened it "permeability" (see § 82).

If the magnetic circuit is not so simply constituted, but is composed of portions of different areas, length, and material in series—as the magnetic circuit of a dynamo is, for instance—the magnetic reluctance can be written (still pursuing the analogy)—

$$R = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \dots,$$

and $I = \frac{4\pi n C}{R}$ as before.

Mutual Induction.

(c) If a single turn of secondary wire surround this closed magnetic circuit, as in Fig. 47, the total induction through it, whatever its shape or size, is just I ; and if it surround the ring n' times, the effective total induction is $n'I$. This is the induction of the primary through the secondary, which, written out in full, is—

$$\frac{4\pi\mu n n' A C}{l}.$$

The relation is a mutual one; and if the same current were to flow in secondary, the same number of lines would thread effectively the primary. Hence we call it *mutual induction*, and write it MC ; where M , the coefficient of mutual induction between the two coils, is—

$$M = \frac{4\pi\mu n n' A}{l};$$

the A and the l referring most easily to the simply and obviously closed magnetic circuit. Two detached coils situated anyhow

with respect to each other, will have a specifiable value of M , but it is not so easy to write down.

Self-Induction.

(*d*) Instead of using a secondary coil to surround the induction caused by the primary, we may consider the primary as surrounding the induction itself has produced, and so speak of its "self-induction" as—

$$\frac{4\pi\mu n^2 AC}{l},$$

which, written LC , gives us the coefficient of self-induction—

$$L = \frac{4\pi\mu n^2 A}{l},$$

or,

$$= 4\pi\mu n_1 A,$$

where n_1 = number of turns per unit length. (§§ 115 and 98)

Here, again, every coil has a specifiable self-induction, but in most cases it is not so easy to write down. It always means, however, the ratio of the self-produced magnetic induction to the current which has produced it—

$$L = \frac{I}{C}.$$

Value of Coefficient of Self-Induction in a few other Simple Cases.

(*e*) The magnetic field produced by a straight wire varies inversely with the distance; being, at a distance r from a straight wire of sectional radius a , conveying a current, C —

$$\frac{2\mu C}{r}.$$

and this therefore specifies the number of lines through unit area.

So the whole number of lines of force included between the wire and any distance b , in a drum of thickness l , is—

$$\int_a^b \frac{2\mu Cl}{r} dr = 2\mu Cl \cdot \log \frac{b}{a}.$$

Now, if at the distance b there is a parallel wire, conveying the return current, it, too, will have the same number of lines of force, and the whole number lying between a length, l , of each of the two parallel wires is—

$$4\mu l \log \frac{b}{a} \times C;$$

and as all the lines of force that exist pass between the wires, this expression sums up the whole magnetic flux produced by the going and return parallel currents; and the coefficient of C in the last expression is therefore the coefficient of self-induction for the case of two thin parallel wires at a distance b .

For a circular loop of radius r , radius of section of wire being a , this modifies itself to—

$$L = 4\pi\mu r \log \frac{8r}{a}$$

(see § 140). In every case μ refers to the space near the wire, not to the substance of the wire itself.

In both these cases, the magnetization of the substance of the wires themselves is supposed *nil*. In the case of extremely rapidly alternating currents, this is correct (§ 47). In the case of copper wires not too close together, it is never *very* incorrect.

Energy of a Current.

(*f*) A magnet of moment ml , in a magnetic field of intensity H , experiences a couple $mlH \sin \theta$; and therefore a simple

stiff coil of wire conveying a current experiences a couple $\mu n A C H \sin \theta$. If it turns a small angle, $d\theta$, the work done, or the change of potential energy, is $\mu n A C H \sin \theta d\theta$; and therefore the potential energy of the circuit in any position is $-\mu n A C H \cos \theta$; which may be written IC , because $nA \cos \theta$ is the effective area of the coil resolved perpendicularly to the lines of force which thread it to the number μH per unit area.

This result may be generalized; a current in a magnetic field always possesses energy IC . If the field is due to external causes, *i.e.* having an existence independent of the current, the energy is potential energy of strain, and tends to cause the circuit to rotate. This is the principle of electric motors. But if the field is due to nothing but the current itself—if it is a self-produced and self-maintained field—the value of I is LC , and the energy is now more conveniently called kinetic energy. To obtain its value, we must remember that the induction and the current die out together: it is not as if they had an independent existence, and so the energy is—

$$\int_0^C I dC = \frac{1}{2} LC^2.$$

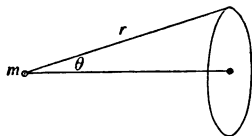
This is the work which must be done at starting and at stopping the current (Chap. V.).

Pole near a Circuit.

(*g*) If a single pole find itself on the axis of a circle, the number of its lines of force which penetrate the circle is $\frac{m}{r^2} \cdot 2\pi r^2(1 - \cos \theta)$, the latter factor being the area of the portion of a sphere with centre at m , cut off by the said circle. The expression $2\pi(1 - \cos \theta)$, since it measures the ratio of the area subtended by a conical angle to the square of the radius, is, in analogy with the circular measure of a plane angle, called a solid

angle: the solid angle of the cone with vertex m and base the circle, or the angle subtended by the circle to an eye placed at m . Call this angle ω ; then the number of lines of force, or the magnetic induction through the circle is $m\omega$.

If the circle becomes now a circuit conveying a current C , the system has energy $m\omega C$, and accordingly there will be a tendency to relative motion, the force in any direction being equal to the rate of change of $m\omega C$ per unit distance in that direction.



The potential of the pole on the circuit is $m\omega$; the potential of the circuit on the pole is $C\omega$. If the pole is situated anywhere, and the coil is of any shape, ω can still be specified, but not so easily. If there is a collection of magnets, their potential on a circuit, or induction through it, can be written $\Sigma(m\omega)$.

Magneto-electricity.

(*h*) The fundamental fact of magneto-electricity is that if the induction through a circuit change from any cause whatever, an E.M.F. is set up in the circuit equal to the rate of change of the magnetic induction—

$$e = \frac{dI}{dt}$$

This is not strictly a relation independent of the fundamental fact of electro-magnetism: the two are connected by the law of the conservation of energy. I may indicate this important

fact sufficiently for our present purpose by quoting the conservation of energy, in a form applicable to the case of a circuit conveying a steady current, as—

$$ECdt = RC^2dt + Cdl ;$$

whence

$$RC = E - \frac{dI}{dt},$$

or the resultant E.M.F. consists not only of the E.M.F. applied, but contains also an intrinsic or indirect E.M.F. magnetically excited in the circuit ; this being what Faraday discovered as magneto-electricity.

Various Modes of exciting Induction Currents.

(i) Now I may be made up in a multitude of ways. It may be a component of terrestrial magnetic field, say, $nAH \cos \theta$. It may be caused by magnets in the neighbourhood $\Sigma(m\omega)$. It may be due to induction from some other coil, MC' . It may be due to the current passing in the coil itself, say LC . The total induced E.M.F. is the rate of change of the sum of all these, or—

$$e = \frac{d}{dt} \{ nAH \cos \theta + \Sigma(m\omega) + MC' + LC \} ;$$

and accordingly it may be excited in many ways: by changes in size or shape of coil ; by changing its aspect to the field (as in a dynamo) ; by moving magnets in its neighbourhood (as in an alternating-current machine) ; by varying the current in or shifting the position of other circuits (as in a Ruhmkorff coil) ; or, lastly, by changing its own current, or its own coefficient of self-induction. Changes in the last term, $\frac{d}{dt}(LC)$, are specially called E.M.F. of self-induction, and used to be called extra currents.

Primary Current alone : and Coil with Revolving Commutator.

(j) The equation to a current of varying strength in the simplest case of a lone circuit is—

$$E - RC = \frac{d}{dt}(LC),$$

where E is the applied E.M.F. ; and this may be written out more fully—

$$L \frac{dC}{dt} + \left(R + \frac{dL}{dt} \right) C = E,$$

which shows that in the case of circuits of variable self-induction the resistance has not its most simple value, but has an extra term in it, a spurious or imitative resistance, $\frac{dL}{dt}$.

An example of a circuit of variable self-induction is one which is continually having wire withdrawn from or added to it, so that a current has to be stopped in portions where it was already established, and started in hitherto stagnant portions : a case quite analogous to the viscosity of gases, and commonly illustrated by passengers of appreciable inertia getting in and out of a moving train. An instance of the case occurs in every Gramme ring, or indeed every dynamo armature, when spinning with a commutator, quite independently of the magnetic field in which it may happen to be spinning. In all such cases the effective resistance is rather greater than R, being $R + \frac{dL}{dt}$, or $R + nL$; where the self-induction virtually added to the circuit n times a second is L.

Leyden Jar.

(k) In the case of a discharging condenser of capacity S, the quantity stored in it at any instant is such that $C = - \frac{dQ}{dt}$,

or that $Q = Q_0 - \int_0^t C dt$; and the difference of potential between its terminals is $\frac{Q}{S}$, which is the E.M.F. applied to the circuit. So the equation to the discharge current is—

$$L \frac{dC}{dt} + RC = \frac{Q}{S}.$$

The solution of the equation in this case is—

$$C = \frac{E}{\rho L} e^{-mt} \sin \phi t,$$

where $m = \frac{R}{2L}$, and regulates the total duration of the discharge,

and where $\phi = \frac{1}{\sqrt{LS}}$ approximately

$$\left\{ \text{more accurately } \sqrt{\left(\frac{1}{LS} - m^2\right)} \right\},$$

and regulates the rapidity of alternation, which is $\frac{\phi}{2\pi}$. The wave-length of the emitted radiation (Chapter XIV.) is—

$$\lambda = \frac{2\pi}{\phi} \cdot v = 2\pi \sqrt{\left(\frac{L}{\mu} \cdot \frac{S}{K}\right)}.$$

With these quick oscillations, R is nothing at all like its ordinary value for steady currents; because the outside of the wire only is used (§§ 45 and 102); but, calling the ordinary value R_0 , R is very approximately, for high rates of alternation,—¹

$$R = \sqrt{\left(\frac{1}{2} \rho \mu_0 l \cdot R_0\right)},$$

l being the length of the wire, and μ_0 the magnetic permeability of its substance (§ 46).

¹ See Rayleigh, *Phil. Mag.*, May 1886.

The emission of radiation by such a circuit goes to increase R still more (§ 142 and p. 367). See also m .

Alternating Current.

(l) In case of any coil or armature spinning in a magnetic field, the equation to the current is—

$$-RC = \frac{d}{dt}(nAH \cos \theta + LC), \quad 1$$

$$\text{or } L \frac{dC}{dt} + \left(R + \frac{dL}{dt}\right)C = nAH \sin \theta \frac{d\theta}{dt};$$

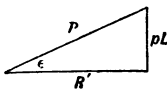
and the E.M.F. is therefore alternating according to a sine function. Writing this equation—

$$L \frac{dC}{dt} + R'C = E_0 \sin \phi t,$$

the solution is—

$$C = \frac{E_0 \cos(\phi t - \epsilon)}{\sqrt{\{R'^2 + (\phi L)^2\}}}$$

where $\tan \epsilon = \frac{\phi L}{R'}$. The R' differs from simple R , as already explained in (j), only when a commutator is employed: which it often is not. The denominator of the above expression



may be called impédance, and denoted by P (see next section), the quantities being related as in this little diagram. The quantity ϵ is the lag of the current behind the applied E.M.F.

Two Definitions of Electric Resistance, and Distinction between the Two.

(*m*) The oldest definition of the term "resistance of a conductor" is that given by Ohm, viz. the ratio—

$$\frac{\text{E.M.F. applied to the conductor.}}{\text{Current excited in it}}$$

But another is contained in the law of Joule, viz. the ratio—

$$\frac{\text{Energy dissipated per second by the conductor.}}{\text{Current squared which it transmits}}$$

In cases of no reversible obstruction the two definitions agree, but in cases of chemical action, of reversible heat effects, and of varying magnetic induction, some of the energy may be stored, all is not dissipated, and under these circumstances the two definitions do not agree. A distinction must be drawn between them: the term resistance cannot properly be applied to both quantities.

Now it is found convenient to retain the name resistance for the second definition—the dissipation of energy coefficient; and to realize that in the total obstruction specified by the first definition there is included "back E.M.F.," "polarization," or other reversible obstruction, in addition to resistance proper; while in the very important case of the total obstruction met with by an alternating current, it has become convenient to call the quantity defined by the first of the two equations, "impédance."

The two definitions of resistance may indeed be always made to agree, if, in the Ohm's law definition, instead of *applied* E.M.F., we reckon *resultant* E.M.F. And this is the neatest and simplest mode of taking into account such things as chemical or thermal polarization, and also a magnetic back E.M.F., so long as it is steady and external, as in the case of electric motors. But, when dealing with alternating generators, some understanding has to be come to as to how the value of their

E.M.F. is to be reckoned, and no simple subtraction of a back E.M.F. is convenient. Referring to last section, we see that the expression for current contains as numerator a lessened or lagging E.M.F., and as denominator an obstruction or impedance containing a term in addition to what is usually called resistance. It is from this point of view that the idea and term "impedance" become so useful.

The value of this quantity is, in general, as has been shown,

$$\sqrt{\{(\rho L)^2 + R^2\}};$$

and its two portions may be styled respectively the inertia, or conservative portion, and the frictional or dissipative portion (§ 38).

Part of the energy dissipated appears as heat in the conductor, and this is the only portion on which Joule experimented, but another portion we now know is propagated out as radiation into space (§ 142): both portions together are included in the numerator proper to the second definition of R.

Induced Current in Secondary Circuit. Transformers.

(n) The E.M.F. induced in a secondary circuit surrounding a ring like Fig. 47, whose primary coil has an alternating or intermittent current, C, sent round it, is, referring back to (h) and (c)—

$$M \frac{dC}{dt}, \quad \text{or} \quad 4\pi n n' \frac{\mu A}{l} \cdot \frac{dC}{dt};$$

and depends, therefore, directly on the number of turns of wire in the secondary coil, and on the rate of variation of the primary current. This is the principle of induction-coils, and of "secondary-generators" or transformers (§ 115). The E.M.F. thus obtained is completely under control by choosing a suitable value for n' , according as high E.M.F. (in Ruhmkorff coils) or a powerful current (for electric welding) is required. They

are called transformers, because, of the two electrical factors in mechanical "power," EC , they can change their ratio, leaving the product nearly constant; just as ordinary machines do with the force and velocity factors of the same product "power." So, in precise analogy with gaining in force what you lose in speed, you gain in E.M.F. what you lose in current; or *vice versa*.

The equations to primary and secondary currents, C and C' are—

$$E - RC = \frac{d}{dt} (LC + MC'),$$

$$0 - R'C' = \frac{d}{dt} (L'C' + MC);$$

and from the solution of these, the effective or apparent self-induction of primary, when its secondary is short-circuited and when all resistances are kept small, comes out equal to $L - \frac{M^2}{L'}$. Now since, for a simply closed magnetic circuit,

$$L : L' : M = n^2 : n'^2 : nn',$$

the effective self-induction (and therefore the impédance) of the primary is approximately zero when its secondary is short-circuited—a fact which is the Magna Charta of commercial transformers.

Rate of Transmission of Telegraph Signals, in the Simplest Case.

(*o*) Consider a unit length of a pair of parallel thin copper wires not very close together, a going and return wire, at a distance b apart, the sectional radius of each wire being a . The self-induction of this portion, see (*e*), is—

$$L_1 = 4\mu \log \frac{b}{a},$$

and the static capacity of the same portion is (by somewhat similar reasoning)—

$$S_1 = \frac{K}{4 \log \frac{b}{a}}.$$

Hence

$$L_1 S_1 = \mu K.$$

The resistance of the same unit length may be called R_1 .

Now consider an element of the pair of wires of length dx , and write down the slope of potential between its ends when a current, C , flows along it, and also their rise of potential with time; we get—

$$\left. \begin{aligned} L_1 \frac{dC}{dt} + R_1 C + \frac{dV}{dx} &= 0, \\ S_1 \frac{dV}{dt} + \frac{dC}{dx} &= 0. \end{aligned} \right\}$$

and

Now, a "wave" being any disturbance periodic both in space and time, its general fundamental equation is—

$$y = a \sin(\phi t - nx),$$

where y is the extent of the disturbance at any place distant x from the origin, and at any time, t , from the era of reckoning.

The coefficient a is the amplitude of the vibration; n is the space-period-constant, or $\frac{2\pi}{\lambda}$; ϕ is the time-period-constant, or

$\frac{2\pi}{T}$; the velocity of advance of the waves is one space-period in

one time-period, viz. $\frac{\lambda}{T}$ or $\frac{\phi}{n}$.

The solution of these equations for the case of an applied rapidly alternating E.M.F., $V \sin \phi t$, at the origin, may be written—

$$V = V_0 e^{-\frac{m_1}{\rho_1} x} \sin \phi \left(t - \frac{x}{\rho_1} \right),$$

where $m_1 = \frac{R_1}{2L_1}$ and $\rho_1 = \frac{1}{\sqrt{(L_1 S_1)}}$.

Hence the above bracketed pair of equations give waves travelling along the wires with the speed $\frac{1}{\sqrt{(L_1 S_1)}}$, which we have seen equals $\frac{1}{\sqrt{(\mu K)}}$, and with an amplitude dying out along the length of the wires according to a logarithmic decrement $\frac{1}{2} R_1 \sqrt{\left(\frac{S_1}{L_1}\right)}$.

The speed of propagation of pulses along wires is therefore precisely the same, in this simple case, as the propagation of waves out through free space, viz. the velocity $\frac{1}{\sqrt{(\mu K)}}$ (§§ 128, 132, 137). All complications go to decrease, not to increase, the speed (§ 135).

Dimensions of Electrical Quantities.

(*p*) Writing L, M, T, F, *v*, for units of length, mass, time, orce, velocity, as usual, and A for area; the fundamental and certain experimental relations, independent of all considerations about units and systems of measurement, are—

$$\text{Of electrostatics,} \quad Q = L \sqrt{(KF)} \quad \dots \quad (1)$$

$$\text{Of magnetism,} \quad m = L \sqrt{(\mu F)} \quad \dots \quad (2)$$

$$\text{Of electro-magnetism,} \quad mL = \mu AC \quad \dots \quad (3)$$

The last may also be written—

$$m = \mu v Q \quad \dots \quad (3')$$

in which form it suggests the magnetic action of a moving charge, which Rowland's experiment has established.

Combining the three equations, we deduce—

$$\sqrt{\left(\frac{\mu}{K}\right)} = \frac{m}{Q} = \mu v;$$

whence

$$\mu K = \frac{1}{v^2} = \frac{\text{density}}{\text{elasticity}},$$

the well-known relation connecting the two ethereal constants.

Comparing many electrical equations with corresponding mechanical ones, we find that the product LC takes the

place of momentum (mv), and that $\frac{1}{2}LC^2$ takes the place of kinetic energy ($\frac{1}{2}mv^2$), and indeed *is* the energy of a current, see (\mathcal{F}). Hence it is natural to think of L as involving inertia, and of μ or $4\pi\mu$ as a kind of density of the medium concerned.

Assuming this, $\frac{4\pi}{K}$ at once becomes an elasticity coefficient (as indeed electrostatics itself suggests), because $\mu K v^2 \equiv 1$; and the dimensions of all electrical units can be specified as follows, without any arbitrary convention or distinction between electrostatic and electro-magnetic units:—

$$\text{Sp. ind. cap., } K = \frac{\text{strain}}{\text{stress}} = \frac{\text{area}}{\text{force}} = \frac{LT^2}{M} = \text{shearability.}$$

$$\text{Permeability, } \mu = \frac{\text{inertia}}{\text{volume}} = \frac{M}{L^3} = \text{density.}$$

$$\text{Electric charge, } Q = L^2 = \frac{\text{volume}}{\text{displacement}}.$$

$$\text{Magnetic pole, } m = \frac{M}{T} = \text{momentum per unit length.}$$

$$\text{Electric current, } C = \frac{L^2}{T} = \text{displacement} \times \text{velocity.}$$

$$\text{Magnetic moment, } ml = \frac{ML}{T} = \text{momentum.}$$

$$\text{E.M.F., } E = \frac{\text{work}}{Q} = \frac{M}{T^2} = \text{pressure} \times \text{displacement, or} \\ \text{work per unit area.}$$

$$\text{Intensity of magnetic field, } H = \frac{F}{m} = \frac{L}{T} = \text{velocity.}$$

$$\text{Intensity of electrostatic field, } \frac{F}{Q} = \frac{M}{LT^2} = \text{energy per unit} \\ \text{volume.}$$

$$\text{Surface density, } \sigma = \frac{Q}{A} = \text{a pure number.}$$

$$\text{Electric tension, } \frac{2\pi\sigma^2}{K} = \frac{M}{LT^2} = \text{a pressure or tension.}$$

Capacity, $S = \frac{Q}{E} = \frac{L^2 T^2}{M} =$ displacement per unit pressure.

Coefficient of resistance, $\frac{E}{C} = \frac{M}{L^2 T} =$ impulse or momentum
per unit volume.

Magneto-motive force, $4\pi nC = \frac{L^3}{T} =$ current.

Reluctance, $\frac{l}{\mu A} = \frac{L^2}{M} = \frac{\text{area}}{\text{inertia}}.$

Magnetic induction, $I = \frac{M}{T} =$ moment of momentum per
unit area.

Coefficient of induction (self or mutual), $\frac{I}{C} = \frac{M}{L^2} =$ inertia per
unit area.

This is, or may be, an improvement on the rough practical system which assumes as of no dimensions sometimes K , and sometimes μ , according as one is dealing with electrostatics or with magnetism; but very likely it is only a stepping-stone. Prof. Fitzgerald has recently suggested that, regarding everything from the strictly kinematic and etherial point of view, both K and μ may be a *slowness* of the vorticity; and by that assumption also everything becomes simple and of unique dimensions. Whatever of this turns out true, it is not to be supposed that we can long go on with two distinct systems of units, the electrostatic and the electromagnetic, and two distinct sets of dimensions for the same quantities; knowing as we do that neither set can by any reasonable chance turn out to be the right one.

NEWTON'S GUESSES CONCERNING THE ETHER.

(*q*) Newton's queries at the end of his "Opticks" finish in the early editions with Query 16, and I have found it difficult to

come across the later queries except in Latin. I therefore here copy such portions of these queries as have an obvious bearing on our present subject, in order to make them more easy of reference.

“*Qu.* 17. If a Stone be thrown into stagnating Water, the Waves excited thereby continue some time to arise in the place where the Stone fell into the Water, and are propagated from thence in concentrick Circles upon the Surface of the Water to great distances. And the Vibrations or Tremors excited in the Air by percussion, continue a little time to move from the place of percussion in concentrick Spheres to great distances. And in like manner, when a Ray of Light falls upon the Surface of any pellucid Body, and is there refracted or reflected, may not Waves of Vibrations or Tremors be thereby excited in the refracting or reflecting Medium at the point of Incidence . . . ?”

“*Qu.* 18. If in two large tall cylindrical Vessels of Glass inverted, two little Thermometers be suspended so as not to touch the Vessels, and the Air be drawn out of one of these Vessels, and these Vessels thus prepared be carried out of a cold place into a warm one; the Thermometer *in vacuo* will grow warm as much and almost as soon as the Thermometer which is not *in vacuo*. And when the Vessels are carried back into the cold place, the Thermometer *in vacuo* will grow cold almost as soon as the other Thermometer. Is not the Heat of the warm Room conveyed through the Vacuum by the Vibrations of a much subtiler Medium than Air, which after the Air was drawn out remained in the Vacuum? And is not this Medium the same with that Medium by which Light is refracted and reflected, and by whose Vibrations Light communicates Heat to Bodies,¹ and is put into Fits of easy Reflexion

¹ Note the precision and propriety of this phrase: far superior to most of the writing on the subject of absorption of radiation during the present century. It could only be improved by substituting *generates in* for “communicates to,” in accordance with the modern kinetic theory of heat.

and easy Transmission? And do not the Vibrations of this Medium in hot Bodies contribute to the intenseness and duration of their Heat? And do not hot Bodies communicate their Heat to contiguous cold ones, by the Vibrations of this Medium propagated from them into the cold ones? And is not this Medium exceedingly more rare and subtile than the Air, and exceedingly more elastick and active? And doth it not readily pervade all bodies? And is it not (by its elastick force) expanded through all the Heavens?"

"*Qu.* 19. Doth not the Refraction of Light proceed from the different density of this Ætherial Medium in different places, the Light receding always from the denser parts of the Medium? And is not the density thereof greater in free and open Space void of Air and other grosser Bodies, than within the Pores of Water, Glass, Crystal, Gems, and other compact Bodies?"¹ . . .

"*Qu.* 21. Is not this medium much rarer in the denser Bodies of the Sun, Stars, Planets, and Comets, than in the empty celestial Spaces between them? And in passing from them to great distances, doth it not grow denser and denser perpetually, and thereby cause the gravity of those great Bodies towards one another, and of their parts towards the Bodies; every body endeavouring to go from the denser parts of the Medium towards the rarer? For if this Medium be rarer within the Sun's Body than at its surface, and rarer there than at the hundredth part of an Inch from its Body, and rarer there than at the fiftieth of an Inch from its Body,² and rarer there than at

¹ In Newton's opinion light travelled quicker in gross matter than in space, and hence it is that he inverts our Fresnel-derived views. He continues the same inversion in his query concerning gravitation, here next following.

² It was his experiments in diffraction which made him think of this gradual change in the properties of ether as one recedes from a body. A few years ago such gradual changes would have seemed to us quite unlikely; but the most recent experiments of Michelson shake all preconceived opinions.

the Orb of *Saturn*; I see no reason why the Increase of density should stop anywhere, and not rather be continued through all distances from the Sun to *Saturn*, and beyond. And though this Increase of density may at great distances be exceeding slow, yet if the elastick force¹ of the medium be exceeding great, it may suffice to impel Bodies from the denser parts of the Medium towards the rarer, with all that power which we call Gravity. And that the elastick force of the Medium is exceeding great, may be gathered from the swiftness of its Vibrations. Sounds move about 1140 English Feet in a second Minute of Time, and in seven or eight Minutes of Time they move about one hundred English Miles. Light moves from the Sun to us in about seven or eight Minutes of Time, which distance is about 70,000,000 English Miles, supposing the horizontal Parallax of the Sun to be about 12". And the Vibrations or Pulses of this Medium, that they may cause the alternate Fits of easy Transmission and easy Reflexion, must be swifter than Light, and by consequence above 700,000 times swifter than Sounds. And therefore the elastick force of this Medium, in proportion to its density, must be above $700,000 \times 700,000$ (that is, above 490,000,000,000) times greater than the elastick force of Air is in proportion to its density. For the Velocities of the Pulses of Elastick Mediums are in a subduplicate Ratio of the Elasticities and the Rarities of the Mediums taken together." . . .

"*Qu.* 22. May not Planets and Comets, and all gross Bodies, perform their motions more freely, and with less resistance in this Ætherial Medium than in any Fluid, which fills all Space adequately without leaving any Pores, and by consequence is much denser than Quick-silver and Gold? And may not its resistance be so small as to be inconsiderable? For instance ;

¹ Meaning what we call the pressure. This is, of course, pursuing the analogy of sound-waves, and does not accord with our present knowledge.

if this Æther (for so I will call it¹) should be supposed 700,000 times more elastick than our Air, and above 700,000 times more rare ; its resistance would be above 600,000,000 times less than that of Water. And so small a resistance would scarce make a sensible alteration in the Motions of the Planets in ten thousand Years. If any one would ask me how a Medium can be so rare, let him tell me how the Air in the upper parts of the Atmosphere can be above an hundred thousand times rarer than Gold. Let him also tell me how an electrick Body can by Friction emit an Exhalation so rare and subtile, and yet so potent, as by its Emission to cause no sensible Diminution of the weight of the electrick Body, and to be expanded through a Sphere whose Diameter is above two Feet, and yet to be able to agitate and carry up Leaf Copper, or Leaf Gold, at the distance of above a Foot from the electrick Body? And how the Effluvia of a Magnet can be so rare and subtile, as to pass through a Plate of Glass without any Resistance or Diminution of their Force, and yet so potent as to turn a magnetick Needle beyond the Glass?"

¹ The interest of these extracts lies largely in their belonging to the very early days of the conception of an ether, and in their remarkable insight into many things, though in detail they often do not completely accord with present knowledge.